Renormalisability of the SU(2)×U(1) Electroweak Theory with Massive W Z Fields and Massive Matter Fields

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Abstract

We extend the previous work and study the renormalisability of the $\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ electroweak theory with massive W Z fields and massive matter fields. We expound that with the constraint conditions caused by the W Z mass term and the additional condition chosen by us we can still performed the quantization in the same way as before. We also show that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields is BRST invariant. Furthermore, with the help of the the renormalisability of the theory without the the mass term of matter fields, we find the general form of the divergent part of the generating functional for the regular vertex functions and prove the renormalisability of the theory with the mass terms of the W Z fields and the matter fields.

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I. Introduction

Owing to the lack of experimental evidence for the Higgs Bosons and to the unsatisfying treatment on quantization the non-Abelian theory with massive gauge fields has been reinvestigated [1-9]. Particularly, it has been clarified [1-3] that the SU(n) theory with massive gauge fields and the SU(2)×U(1) theory of S.L.Glashow [10] with massive W Z fields are renormalisable. In the present paper we will extend these work and study the renormalisability of the latter electroweak theory with the massive W Z fields and massive matter fields. For the sake of convenience we assume that the matter fields consist only of the electron and electron-neutrino fields.

With the W Z mass term and the matter field mass term directly added to the Lagrangian by hand, the classical equations of motion will yield complicated constraint conditions containing products of the field functions. Moreover, when the constraints are expressed so that the gauge field parts contain no mass parameters the matter field parts will have a negative dimension coefficient m/M^2, where M and m are the mass parameters of the W fields and the electron fields respectively.

As in the case of Ref. [3], since the mass term is invariant under an infinitesimal gauge transformation with $\delta\theta_1$ and $\delta\theta_2$ equal to zero and $\delta\theta_3$ equal to $\delta\theta_y$, where θ_a and θ_1 are the parameters of the gauge group, an additional constraint condition should be properly chosen. We will expound that with the constraint conditions caused by the mass term and the additional condition chosen by us we can performed the quantization and construct the ghost action in a way similar to that used in Refs. [1,3]. We will also show that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields is BRST invariant. A special thing is that the effective action has a matter–ghost term coming from the matter field parts of the constraint conditions and containing the factor m/M^2.

We will follow the procedure of Ref. [3] and use the generalized form of the theory containing λ_a, λ_y and their sources in the generating functional for the Green functions to study the renormalisability of the theory containing only the original fields and the ghost fields. Namely, after deriving the Slavnov–Taylor identities and the additional identities for the generating functional Γ for the regular vertex functions with the help of the generalized form of the theory, we will let vanish the functional derivatives of Γ with respect to the classical fields of these Lagrange multipliers. In this way the divergent part of Γ will be shown to satisfy a set of equations which can still be treated. Furthermore, with the help of the the renormalisability of the theory without the the mass term of matter fields, we will be able to find the

general form of the divergent part of Γ and prove that the mass term of the matter fields is also harmless to the renormalisability of the theory.

In spite of the extra complexeity caused by the mass term of the matter fields we will write this paper in the similar form as that of Ref. [3]. In section 2 we will find the constraint conditions coming from the W Z mass term and choose the additional constraint condition. The method of quantization will be explained in section 3. Setion 4 is devoted to prove the renormalisability of the theory. Concluding remarks will be given in the final section.

II. Original and Additional Constraint Conditions

The matter fields will be often denoted by $\psi(x)$ and $\overline{\psi}(x)$ and they only contain the electron fields and electron-neutrino fields in the present work. The former stands for the purely left-handed neutrino field ν_L, the left- and right-handed parts of the electron field namely e_L, e_R, and the latter stands for $\overline{\nu}_L$, $\overline{e}_L$ and $\overline{e}_R$. Therefore the mass term of the matter fields is

$$\mathcal{L}_{\psi m}(x) = -m\overline{e}_L(x)e_R(x) - m\overline{e}_R(x)e_L(x)\,. \tag{2.1}$$

Next let $W_{a\mu}(x)$, $W_{y\mu}(x)$ be the $\mathrm{SU}_L(2)$ and $\mathrm{U}_Y(1)$ gauge fields and g, g_1 be the coupling constants. Thus the W Z mass term in the Lagrangian is

$$\mathcal{L}_{WM} = \frac{1}{2}M^2 W_{a\mu}W_a^\mu + \frac{1}{2}M^2\Big(\frac{g_1}{g}\Big)^2 W_{y\mu}W_y^\mu - M^2\Big(\frac{g_1}{g}\Big)W_{3\mu}W_y^\mu\,, \tag{2.2}$$

or

$$\mathcal{L}_{WM} = \frac{1}{2}M^2 W_{1\mu}(x)W_1^\mu(x) + \frac{1}{2}M^2 W_{2\mu}(x)W_2^\mu(x) + \frac{1}{2}M_z^2 Z_\mu(x)Z^\mu(x)\,,$$

where M_z^2 stands for $g^{-2}(g^2+g_1^2)M^2$, and $Z_\mu(x)$, $A_\mu(x)$ are the field functions of Z boson and photon, namely

$$Z_\mu = \frac{1}{\sqrt{(g^2+g_1^2)}}(gW_{3\mu} - g_1 W_{y\mu})\,, \tag{2.3}$$

$$A_\mu = \frac{1}{\sqrt{(g^2+g_1^2)}}\varepsilon(g_1 W_{3\mu} + gW_{y\mu})\,, \tag{2.4}$$

where ε is 1 or -1.

The original Lagrangian of the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ electroweak theory with the mass term $\mathcal{L}_{WM}$ is

$$\mathcal{L} = \mathcal{L}_{\psi m} + \mathcal{L}_\psi + \mathcal{L}_{\psi W} + \mathcal{L}_{WM} + \mathcal{L}_{WL} + \mathcal{L}_{WY}\,, \tag{2.5}$$

where $\mathcal{L}_\psi$ describe the pure matter fields, $\mathcal{L}_{\psi W}$ is the coupling term between the matter and gauge fields. $\mathcal{L}_{WL}$ and $\mathcal{L}_{WY}$ are the gauge field parts without mass terms, namely

$$\mathcal{L}_{WL} = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu}\,, \tag{2.6}$$

$$\mathcal{L}_{WY} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,, \tag{2.7}$$

where

$$F_{a\mu\nu} = \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - gC_{abc}W_{b\mu}W_{c\nu}\,, \tag{2.8}$$

$$B_{\mu\nu} = \partial_\mu W_{y\nu} - \partial_\nu W_{y\mu}\,. \tag{2.9}$$

C_{abc} stands for the structure constants of $\mathrm{SU}_L(2)$ with C_{123} equal to 1.

Denote by $\theta_a(x), \theta_y(x)$ the parameters of the gauge group. Thus, under an infinitesimal gauge transformation, the fields W_a^μ, W_y^μ, ψ and $\overline{\psi}$ transform as

$$\begin{aligned}
\delta W_a^\mu(x) &= -\frac{1}{g}\partial^\mu\delta\theta_a(x) - C_{abc}W_c^\mu(x)\delta\theta_b(x)\,,\\
\delta W_y^\mu(x) &= -\frac{1}{g_1}\partial^\mu\delta\theta_y(x)\,,\\
\delta\nu_L(x) &= \frac{i}{2}\delta\theta_1(x)e_L(x) + \frac{1}{2}\delta\theta_2(x)e_L(x) + \frac{i}{2}\delta\theta_3(x)\nu_L(x) - \frac{i}{2}\delta\theta_y(x)\nu_L(x)\,,\\
\delta e_L(x) &= \frac{i}{2}\delta\theta_1(x)\nu_L(x) - \frac{1}{2}\delta\theta_2(x)\nu_L(x) - \frac{i}{2}\delta\theta_3(x)e_L(x) - \frac{i}{2}\delta\theta_y(x)e_L(x)\,,\\
\delta e_R(x) &= -i\delta\theta_y(x)e_R(x)\,,\\
\delta\overline{\nu}_L(x) &= -\frac{i}{2}\delta\theta_1(x)\overline{e}_L(x) + \frac{1}{2}\delta\theta_2(x)\overline{e}_L(x) - \frac{i}{2}\delta\theta_3(x)\overline{\nu}_L(x) + \frac{i}{2}\delta\theta_y(x)\overline{\nu}_L(x)\,,\\
\delta\overline{e}_L(x) &= -\frac{i}{2}\delta\theta_1(x)\overline{\nu}_L(x) - \frac{1}{2}\delta\theta_2(x)\overline{\nu}_L(x) + \frac{i}{2}\delta\theta_3(x)\overline{e}_L(x) + \frac{i}{2}\delta\theta_y(x)\overline{e}_L(x)\,,\\
\delta\overline{e}_R(x) &= i\delta\theta_y(x)\overline{e}_R(x)\,.
\end{aligned}$$

$\delta\mathcal{L}_{\psi m}$ can be written as

$$\delta\mathcal{L}_{\psi m} = f_a(x)\delta\theta_a(x) + f_y(x)\delta\theta_y(x)\,, \tag{2.10}$$

where

$$f_1(x) = \frac{i}{2}m\Big\{\overline{\nu}_L(x)e_R(x) - \overline{e}_R(x)\nu_L(x)\Big\}\,, \tag{2.11}$$

$$f_2(x) = \frac{1}{2}m\Big\{\overline{\nu}_L(x)e_R(x) + \overline{e}_R(x)\nu_L(x)\Big\}\,, \tag{2.12}$$

$$f_3(x) = \frac{i}{2}m\Big\{\overline{e}_R(x)e_L(x) - \overline{e}_L(x)e_R(x)\Big\}\,, \tag{2.13}$$

$$f_y(x) = -f_3(x)\,. \tag{2.14}$$

Therefore the action transforms as

$$
\begin{aligned}
\delta\int d^4x\mathcal{L}(x) &= \delta\int d^4x\Big\{\mathcal{L}_{WM}(x)+\mathcal{L}_{\psi m}(x)\Big\}\\
&= \int d^4x\Big\{\Big(\frac{M^2}{g}\partial_\mu W_1^\mu(x)+\frac{M^2}{g}g_1W_{2\mu}(x)W_y^\mu(x)+f_1(x)\Big)\delta\theta_1\\
&\quad+\Big(\frac{M^2}{g}\partial_\mu W_2^\mu(x)-\frac{M^2}{g}g_1W_{1\mu}(x)W_y^\mu(x)+f_2(x)\Big)\delta\theta_2\\
&\quad+\Big(\frac{M^2}{g}\partial_\mu W_3^\mu(x)-\frac{M^2}{g^2}g_1\partial_\mu W_y^\mu(x)+f_3(x)\Big)(\delta\theta_3-\delta\theta_y)\Big\}\,.
\end{aligned}
\tag{2.15}
$$

Since the classical equations of motion make the action invariant under an arbitrary infinitesimal transformation of the field functions, they certainly make the mass term invariant under an arbitrary infinitesimal gauge transformation. This means that when M is not equal to zero, the classical equations of motion leads to the following constraint conditions

$$\frac{M^2}{g}\partial_\mu W_1^\mu(x)+\frac{M^2}{g}g_1W_{2\mu}(x)W_y^\mu(x)+f_1(x)=0\,, \tag{2.16}$$

$$\frac{M^2}{g}\partial_\mu W_2^\mu(x)-\frac{M^2}{g}g_1W_{1\mu}(x)W_y^\mu(x)+f_2(x)=0\,, \tag{2.17}$$

$$\frac{M^2}{g}\partial_\mu W_3^\mu(x)-\frac{M^2}{g^2}g_1\partial_\mu W_y^\mu(x)+f_3(x)=0\,. \tag{2.18}$$

These are the original constraint conditions. Since the mass term is invariant under an infinitesimal gauge transformation with $\delta\theta_1$ and $\delta\theta_2$ equal to zero and $\delta\theta_3$ equal to $\delta\theta_y$, $\partial_\mu W_3^\mu$ and $\partial_\mu W_y^\mu$ appear in one constraint. We now choose an additional condition and replace (2.18) with

$$\frac{M^2}{g}\partial_\mu W_3^\mu(x)+\frac{M^2}{g}g_1W_{3\mu}(x)W_y^\mu(x)+f_3(x)=0\,, \tag{2.19}$$

$$\partial_\mu W_y^\mu(x)+gW_{3\mu}(x)W_y^\mu(x)=0\,. \tag{2.20}$$

III. Quantization and BRST Invariance

Write (2.16), (2.17) and (2.19),(2.20) as

$$\Phi_a(x)=0\,,\qquad \Phi_y(x)=0\,, \tag{3.1}$$

with

$$\Phi_1(x)=\partial_\mu W_1^\mu(x)+g_1W_{2\mu}(x)W_y^\mu(x)+\frac{g}{M^2}f_1(x)\,, \tag{3.2}$$

$$\Phi_2(x)=\partial_\mu W_2^\mu(x)-g_1W_{1\mu}(x)W_y^\mu(x)+\frac{g}{M^2}f_2(x)\,, \tag{3.3}$$

$$\Phi_3(x)=\partial_\mu W_3^\mu(x)+g_1W_{3\mu}(x)W_y^\mu(x)+\frac{g}{M^2}f_3(x)\,, \tag{3.4}$$

$$\Phi_y(x)=\partial_\mu W_y^\mu(x)+gW_{3\mu}(x)W_y^\mu(x)\,. \tag{3.5}$$

Taking the constraint conditions (3.1) into account one should write the path integral of the Green functions inolving only the original fields as

$$\frac{1}{N_0}\int \mathcal{D}[\mathcal{W},\overline{\psi},\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)W_{a\mu}(x)W_{b\nu}(y)\cdots\exp\{\mathrm{i}I\}\,, \tag{3.6}$$

where

$$\begin{aligned} I &= \int d^4x\mathcal{L}(x)\,, \\ N_0 &= \int \mathcal{D}[\mathcal{W},\overline{\psi},\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)\exp\{\mathrm{i}I\}\,. \end{aligned}$$

Since only the field functions which satisfy the constraint conditions can play roles in the integral (3.6), the value of the Lagrangian can be changed for the field functions which do not satisfy these conditions. In view of the fact that the conditions (3.1) make the action invariant with respect to the infinitesimal gauge trasformation, we now imagine to replace the mass term $\mathcal{L}_{WM}$ in (3.6) with a gauge invariant mass term which is equal to $\mathcal{L}_{WM}$ when the conditions (3.1) are satisfied. Thus, analogous to the case in the Fadeev–Popov method [1,3,11-16], $\Delta[\mathcal{W},\overline{\psi},\psi]$ should be gauge invariant and make the following equation valid for an arbitrary gauge invariant quantity $\mathcal{O}(\mathcal{W},\,\overline{\psi},\,\psi)$

$$\begin{aligned} &\int \mathcal{D}[\mathcal{W},\,\overline{\psi},\,\psi]\Delta[\mathcal{W},\overline{\psi},\psi]\prod_{a',x'}\delta\left(\Phi_{a'}(x')\right)\delta\left(\Phi_y(x')\right)\mathcal{O}(\mathcal{W},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\} \\ &\qquad \propto \int \mathcal{D}[\mathcal{W},\,\overline{\psi},\,\psi]\mathcal{O}(\mathcal{W},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\}\,, \end{aligned}$$

where $\widetilde{I}$ is a gauge invariant action constructed by replacing $\mathcal{L}_{WM}$ with the imagined mass term. This means that the weight factor $\Delta[\mathcal{W},\overline{\psi},\psi]$ can be determined according to the Fadeev–Popov equation of the following form

$$\Delta[\mathcal{W},\overline{\psi},\psi]\int\prod_z d\Omega(z)\prod_{\sigma,x}\delta\Big(\Phi_\sigma^\Omega(x)\Big) = 1\,. \tag{3.7}$$

where σ stands for $1,2,3,y$, $\Phi_\sigma^\Omega(x)$ is the result of acting on $\Phi_\sigma(x)$ with a gauge transformation having the parameters of the element $\Omega(x)$ of the gauge group, $d\Omega(z)$ is the volume element of the group integral. It follows that with the F–P ghost fields $C_a(x)$, $C_y(x)$, $\overline{C}_a(x)$, $\overline{C}_y(x)$ as new variables, one can express the ghost Lagrangian as

$$\mathcal{L}^{(C)}(x) = \overline{C}_a(x)\Delta\Phi_a(x) + \overline{C}_y(x)\Delta\Phi_y(x)\,, \tag{3.8}$$

where $\Delta\Phi_a(x)$, $\Delta\Phi_y(x)$ are defined by the BRST transformtion of $\Phi_a(x)$ and $\Phi_y(x)$ so that

$$\delta_B\Phi_a(x) = \delta\zeta\Delta\Phi_a(x)\,,\qquad \delta_B\Phi_y(x) = \delta\zeta\Delta\Phi_y(x)\,, \tag{3.9}$$

where $\delta\zeta$ is an infinitesimal fermionic parameter independent of x. The BRST transformation of the gauge fields or matter fields is nothing but the infinitesimal gauge transformation with $\delta\theta_a$ and $\delta\theta_y$ equal to $-g\delta\zeta C_a$ and $-g_1\delta\zeta C_y$ respectively. Namely

$$\delta_B W_a^\mu(x) = \delta\zeta\Delta W_a^\mu(x) = \delta\zeta D_{ab}^\mu C_b(x)\,, \tag{3.10}$$

$$\delta_B W_y^\mu(x) = \delta\zeta\Delta W_y^\mu(x) = \delta\zeta\partial^\mu C_y(x)\,, \tag{3.11}$$

$$\delta_B\psi(x) = \delta\zeta\Delta\psi(x)\,, \qquad \delta_B\overline{\psi}(x) = \delta\zeta\Delta\overline{\psi}(x)\,, \tag{3.12}$$

where

$$\begin{aligned}
D_{ab}^\mu(x) &= \delta_{ab}\partial^\mu + gf_{abc}A_c^\mu(x)\,,\\
\Delta\nu_L(x) &= -\frac{i}{2}gC_1(x)e_L(x) - \frac{1}{2}gC_2(x)e_L(x) - \frac{i}{2}gC_3(x)\nu_L(x) + \frac{i}{2}g_1C_y(x)\nu_L(x)\,,\\
\Delta e_L(x) &= -\frac{i}{2}gC_1(x)\nu_L(x) + \frac{1}{2}gC_2(x)\nu_L(x) + \frac{i}{2}gC_3(x)e_L(x) + \frac{i}{2}g_1C_y(x)e_L(x)\,,\\
\Delta e_R(x) &= ig_1C_y(x)e_R(x)\,,\\
\Delta\overline{\nu}_L(x) &= \frac{i}{2}gC_1(x)\overline{e}_L(x) - \frac{1}{2}gC_2(x)\overline{e}_L(x) + \frac{i}{2}gC_3(x)\overline{\nu}_L(x) - \frac{i}{2}g_1C_y(x)\overline{\nu}_L(x)\,,\\
\Delta\overline{e}_L(x) &= \frac{i}{2}gC_1(x)\overline{\nu}_L(x) + \frac{1}{2}gC_2(x)\overline{\nu}_L(x) - \frac{i}{2}gC_3(x)\overline{e}_L(x) - \frac{i}{2}g_1C_y(x)\overline{e}_L(x)\,,\\
\Delta\overline{e}_R(x) &= -ig_1C_y(x)\overline{e}_R(x)\,.
\end{aligned}$$

$C_a(x)$ and $C_y(x)$ are also transformed as usual

$$\begin{aligned}
&\delta_B C_a(x) = \delta\zeta\Delta C_a(x) = \delta\zeta\frac{g}{2}C_{abc}C_b(x)C_c(x)\,,\\
&\delta_B C_y(x) = 0\,.
\end{aligned}$$

Now we can write $\Delta\Phi_a(x)$, $\Delta\Phi_y(x)$ as

$$\Delta\Phi_1 = \partial_\mu\Delta W_1^\mu(x) + g_1\Delta W_2^\mu(x)W_{y\mu}(x) + g_1W_{2\mu}(x)\Delta W_y^\mu(x) + \frac{g}{M^2}\Delta f_1(x)\,, \tag{3.13}$$

$$\Delta\Phi_2 = \partial_\mu\Delta W_2^\mu(x) - g_1\Delta W_1^\mu(x)W_{y\mu}(x) - g_1W_{1\mu}(x)\Delta W_y^\mu(x) + \frac{g}{M^2}\Delta f_2(x)\,, \tag{3.14}$$

$$\Delta\Phi_3 = \partial_\mu\Delta W_3^\mu(x) + g_1\Delta W_3^\mu(x)W_{y\mu}(x) + g_1W_{3\mu}(x)\Delta W_y^\mu(x) + \frac{g}{M^2}\Delta f_3(x)\,, \tag{3.15}$$

$$\Delta\Phi_y = \partial_\mu\Delta W_y^\mu(x) + g\Delta W_3^\mu(x)W_{y\mu}(x) + gW_{3\mu}(x)\Delta W_y^\mu(x)\,, \tag{3.16}$$

where

$$\begin{aligned}
\Delta f_1(x) &= \frac{i}{2}m\Big\{\Big(\Delta\overline{\nu}_L(x)\Big)e_R(x) - \overline{\nu}_L(x)\Delta e_R(x) - \Big(\Delta\overline{e}_R(x)\Big)\nu_L(x) + \overline{e}_R(x)\Delta\nu_L(x)\Big\}\,,\\
\Delta f_2(x) &= \frac{1}{2}m\Big\{\Big(\Delta\overline{\nu}_L(x)\Big)e_R(x) - \overline{\nu}_L(x)\Delta e_R(x) + \Big(\Delta\overline{e}_R(x)\Big)\nu_L(x) - \overline{e}_R(x)\Delta\nu_L(x)\Big\}\,,\\
\Delta f_3(x) &= \frac{i}{2}m\Big\{\Big(\Delta\overline{e}_R(x)\Big)e_L(x) - \overline{e}_R(x)\Delta e_L(x) - \Big(\Delta\overline{e}_L(x)\Big)e_R(x) + \overline{e}_L(x)\Delta e_R(x)\Big\}\,.
\end{aligned}$$

Since ΔW_a^μ, ΔW_y^μ, $\Delta\psi(x)$, $\Delta\overline{\psi}(x)$ and $\Delta C_a(x)$ are BRST invariant, it is easy to see that $\Delta\Phi_a(x)$ and $\Delta\Phi_y(x)$ are also BRST invariant.

One can further generalized the theory by regarding as new variables the Lagrange multipliers $\lambda_a(x)$ and $\lambda_y(x)$ associated with the constraint conditions. Thus the total effective Lagrangian and action consist of these Lagrange multipliers, ghosts and the original variables, namely

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}(x) + \mathcal{L}^{(C)}(x) + \lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x)\,, \tag{3.17}$$

$$I_{\rm eff} = \int d^4x \mathcal{L}_{\rm eff}(x)\,. \tag{3.18}$$

Correspondingly, the path integral of the generating functional for the Green functions is

$$\mathcal{Z}[\overline{\eta},\eta,\overline{\chi},\chi,J,j] = \frac{1}{N_\lambda}\int \mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\exp\Big\{{\rm i}\big(I_{\rm eff}+I_s\big)\Big\}, \tag{3.19}$$

where N_λ is a constant, I_s is the source term in the action. They are defined by

$$\begin{aligned} N_\lambda &= \int \mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\exp\Big\{{\rm i}I_{\rm eff}\Big\}\,, \\ I_s &= \int d^4x\Big\{\overline{\eta}(x)\psi(x) + \overline{\psi}(x)\eta(x) + \overline{\chi}_a(x)C_a(x) + \overline{C}_a(x)\chi_a(x) + \overline{\chi}_y(x)C_y(x) \\ &\quad + \overline{C}_y(x)\chi_y(x) + J_a^\mu(x)W_{a\mu}(x) + J_y^\mu(x)W_{y\mu}(x) + j_a(x)\lambda_a(x) + J_y(x)\lambda_y(x)\Big\}\,, \end{aligned} \tag{3.20}$$

where $\overline{\eta}(x),\eta(x)\cdots$ stand for the sources. In particular, $j_a(x)$, $j_y(x)$ are the sources of $\lambda_a(x)$, $\lambda_y(x)$, respectively.

We now check the BRST invariance of the effective action I_{eff} defined by (3.17) and (3.18). With $\overline{C}_a(x)$, $\overline{C}_y(x)$ transforming as

$$\delta_B\overline{C}_a(x) = -\delta\zeta\lambda_a(x)\,, \qquad \delta_B\overline{C}_y(x) = -\delta\zeta\lambda_y(x)\,.$$

and noticing the invariance of $\Delta\Phi_a, \Delta\Phi_y$, one has

$$\delta_B\int d^4x\mathcal{L}^{(C)}(x) = \int d^4x\Big\{-\lambda_a(x)\delta_B\Phi_a(x) - \lambda_y(x)\delta_B\Phi_y(x)\Big\}\,.$$

Therefore

$$\delta_B I_{\rm eff} = \delta_B\int d^4x\Big\{\mathcal{L}_{WM} + \mathcal{L}_{\psi m}\Big\} + \int d^4x\Big\{\big(\delta_B\lambda_a(x)\big)\Phi_a(x) + \big(\delta_B\lambda_y(x)\big)\Phi_y(x)\Big\}\,.$$

From this and the expression of $\delta_B I_{WM}$, it can be shown that the effective action is invariant, when the transformation of $\lambda_a(x)$ and $\lambda_y(x)$ are defined as

$$\delta_B\lambda_1(x) = \delta\zeta M^2 C_1(x)\,,$$

$$
\begin{aligned}
\delta_B\lambda_2(x) &= \delta\zeta M^2 C_2(x)\,, \\
\delta_B\lambda_3(x) &= \delta\zeta M^2 C_3(x) - \delta\zeta\frac{g_1}{g}M^2 C_y(x)\,, \\
\delta_B\lambda_y(x) &= \delta\zeta\frac{g_1^2}{g^2}M^2 C_y(x) - \delta\zeta\frac{g_1}{g}M^2 C_3(x)\,.
\end{aligned}
$$

IV. Renormalisability

Following the notations of Ref. [3], let $W_{a\mu}(x), W_{y\mu}(x)$, $C_a(x), C_y(x), \cdots$ stand for the renormalized field founctions, g, g_1 and M be renormalized parameters. By introducing the source terms of the composite field functions ΔW_a^μ, ΔW_y^μ, $\Delta C_a(x)$, $\Delta\psi(x)$, $\Delta\overline{\psi}(x)$ and the sources $K_\mu^a(x)$, $K_\mu^y(x)$, $L_a(x)$, $n_\alpha(x)$, $l_\alpha(x)$, $p_\alpha(x)$, $n'_\alpha(x)$, $l'_\alpha(x)$ and $p'_\alpha(x)$, the effective Lagrangian without counterterm becomes

$$
\begin{aligned}
\mathcal{L}_{eff}^{[0]}(x) \;=\;& \lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x) + \mathcal{L}_{WL}(x) + \mathcal{L}_{WY}(x) \\
&+\mathcal{L}_{WM}(x) + \mathcal{L}^{(C)}(x) + \mathcal{L}_\psi(x) + \mathcal{L}_{\psi m}(x) + \mathcal{L}_{\psi W}(x) \\
&+K_\mu^a(x)\Delta W_a^\mu(x) + K_\mu^y(x)\Delta W_y^\mu(x) + L_a(x)\Delta C_a(x) \\
&+n_\alpha(x)\Delta\nu_{L\alpha}(x) + l_\alpha(x)\Delta e_{L\alpha}(x) + p_\alpha(x)\Delta e_{R\alpha}(x) \\
&+n'_\alpha(x)\Delta\overline{\nu}_{L\alpha}(x) + l'_\alpha(x)\Delta\overline{e}_{L\alpha}(x) + p'_\alpha(x)\Delta\overline{e}_{R\alpha}(x)\,.
\end{aligned} \tag{4.1}
$$

The complete effective Lagrangian is the sum of $\mathcal{L}_{eff}^{[0]}$ and the counterterm $\mathcal{L}_{count}$

$$
\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{[0]} + \mathcal{L}_{count}\,. \tag{4.2}
$$

With (4.1), the generating functional for Green functions is defined as

$$
\mathcal{Z}^{[0]}[\overline{\eta},\eta,\overline{\chi},\chi,J,j,K,L,n,l,p,n',l',p'] = \frac{1}{N}\int \mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\exp\Big\{{\rm i}\big(I_{eff}^{[0]} + I_s\big)\Big\}\,, \tag{4.3}
$$

$I_{eff}^{[0]}$ is the effective action $\int d^4x\mathcal{L}_{eff}^{[0]}(x)$, N is a constant to make $\mathcal{Z}^{[0]}$ equal to 1 in the absence of the sources, I_s is the source term

$$
\begin{aligned}
I_s = \int d^4x\Big\{&\overline{\eta}(x)\psi(x) + \overline{\psi}(x)\eta(x) + \overline{\chi}_a(x)C_a(x) + \overline{C}_a(x)\chi_a(x) + \overline{\chi}_y(x)C_y(x) \\
&+\overline{C}_y(x)\chi_y(x) + J_a^\mu(x)W_{a\mu}(x) + J_y^\mu(x)W_{y\mu}(x) + j_a(x)\lambda_a(x) + j_y(x)\lambda_y(x)\Big\}\,,
\end{aligned}
$$

where $\overline{\eta}\,\psi$ and $\overline{\psi}\,\eta$ stand for

$$
\begin{aligned}
\overline{\eta}\,\psi &= \overline{\eta}_\alpha^{(\nu)}\,\nu_{L\alpha} + \overline{\eta}_\alpha^{(l)}\,e_{L\alpha} + \overline{\eta}_\alpha^{(r)}\,e_{R\alpha}\,, \\
\overline{\psi}\,\eta &= \overline{\nu}_{L\alpha}\,\eta_\alpha^{(\nu)} + \overline{e}_{L\alpha}\,\eta_\alpha^{(l)} + \overline{e}_{R\alpha}\,\eta_\alpha^{(r)}\,.
\end{aligned}
$$

Denoting by $\mathcal{W}^{[0]}$ and $\Gamma^{[0]}$ the generating functionals for connected Green functions and regular vertex functions respectively, one has

$$\mathcal{Z}^{[0]} = \exp\Big\{ \mathrm{i}\mathcal{W}^{[0]}[\overline{\eta}, \eta, \overline{\chi}, \chi, J, j, K, L, n, l, p, n', l', p'] \Big\}, \tag{4.4}$$

$$\begin{aligned}
&\Gamma^{[0]}[\widetilde{\psi}, \widetilde{\overline{\psi}}, \widetilde{W}, \widetilde{\overline{C}}, \widetilde{C}, \widetilde{\lambda}, K, L, n, l, p, n', l', p'] \\
&\quad = \mathcal{W}^{[0]} - \int d^4x \Big[J_a^\mu \widetilde{W}_{a\mu} + J_y^\mu \widetilde{W}_{y\mu} + j_a \widetilde{\lambda}_a + j_y \widetilde{\lambda}_y + \overline{\chi}_a \widetilde{C}_a + \widetilde{\overline{C}}_a \chi_a + \overline{\chi}_y \widetilde{C}_y \\
&\qquad + \widetilde{\overline{C}}_y \chi_y + \overline{\eta}^{(\nu)} \widetilde{\nu}_L + \overline{\eta}^{(l)} \widetilde{e}_L + \overline{\eta}^{(r)} \widetilde{e}_R + \widetilde{\overline{\nu}}_L \eta^{(\nu)} + \widetilde{\overline{e}}_L \eta^{(l)} + \widetilde{\overline{e}}_R \eta^{(r)} \Big],
\end{aligned} \tag{4.5}$$

where $\widetilde{W}_{a\mu}$, $\widetilde{\nu}_L$, $\cdots$ are the so-called classical fields defined by

$$\begin{aligned}
&\widetilde{W}_{a\mu}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta J_a^\mu(x)}, \qquad \widetilde{\lambda}_a(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta j_a(x)}, \qquad \widetilde{C}_a(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\chi}_a(x)}, \\
&\widetilde{\overline{C}}_a(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \chi_a(x)}, \qquad \widetilde{W}_{y\mu}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta J_y^\mu(x)}, \qquad \widetilde{\lambda}_y = \frac{\delta \mathcal{W}^{[0]}}{\delta j_y(x)}, \\
&\widetilde{C}_y(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\chi}_y(x)}, \qquad \widetilde{\overline{C}}_y(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \chi_y(x)}, \qquad \widetilde{\nu}_{L\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(\nu)}(x)}, \\
&\widetilde{e}_{L\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(l)}(x)}, \qquad \widetilde{e}_{R\alpha}(x) = \frac{\delta \mathcal{W}^{[0]}}{\delta \overline{\eta}_\alpha^{(r)}(x)}, \qquad \widetilde{\overline{\nu}}_{L\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(\nu)}(x)}, \\
&\widetilde{\overline{e}}_{L\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(l)}(x)}, \qquad \widetilde{\overline{e}}_{R\alpha}(x) = -\frac{\delta \mathcal{W}^{[0]}}{\delta \eta_\alpha^{(r)}(x)},
\end{aligned}$$

Therefore

$$\begin{aligned}
&J_a^\mu(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{W}_{a\mu}(x)}, \qquad j_a(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\lambda}_a(x)}, \qquad \overline{\chi}_a(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{C}_a(x)}, \\
&\chi_a(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)}, \qquad J_y^\mu(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{W}_{y\mu}(x)}, \qquad j_y(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\lambda}_y(x)}, \\
&\overline{\chi}_y(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{C}_y(x)}, \qquad \chi_y(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_y(x)}, \qquad \eta_\alpha^{(\nu)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{\nu}}_{L\alpha}(x)}, \\
&\eta_\alpha^{(l)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{L\alpha}(x)}, \qquad \eta_\alpha^{(r)}(x) = -\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{e}}_{R\alpha}(x)}, \qquad \overline{\eta}_\alpha^{(\nu)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{\nu}_{L\alpha}(x)}, \\
&\overline{\eta}_\alpha^{(l)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{e}_{L\alpha}(x)}, \qquad \overline{\eta}_\alpha^{(r)}(x) = \frac{\delta \Gamma^{[0]}}{\delta \widetilde{e}_{R\alpha}(x)}.
\end{aligned}$$

Besides, for $K_\mu^a, L_a \cdots$, the spectators in the Legendre transtrormation, one has

$$\begin{aligned}
&\frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu^a(x)} = \frac{\delta \Gamma^{[0]}}{\delta K_\mu^a(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta K_\mu^y(x)} = \frac{\delta \Gamma^{[0]}}{\delta K_\mu^y(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta L_a(x)} = \frac{\delta \Gamma^{[0]}}{\delta L_a(x)}, \\
&\frac{\delta \mathcal{W}^{[0]}}{\delta n_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta n_\alpha(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta l_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta l_\alpha(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta p_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta p_\alpha(x)}, \\
&\frac{\delta \mathcal{W}^{[0]}}{\delta n'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta n'_\alpha(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta l'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta l'_\alpha(x)}, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta p'_\alpha(x)} = \frac{\delta \Gamma^{[0]}}{\delta p'_\alpha(x)}.
\end{aligned}$$

In order to find the Slavnov–Taylor identity satisfied by the generating functional for the regular vertex functions, we change the variables in the path integral of $\mathcal{Z}^{[0]}$ as follows

$$
\begin{aligned}
&W_a^\mu(x) \to W_a^\mu(x) + \delta\zeta\Delta W_a^\mu(x)\,, \qquad W_y^\mu(x) \to W_y^\mu(x) + \delta\zeta\Delta W_y^\mu(x)\,,\\
&C_a(x) \to C_a(x) + \delta\zeta\Delta C_a(x)\,, \qquad C_y(x) \to C_y(x)\,,\\
&\overline{C}_a(x) \to \overline{C}_a(x) - \delta\zeta\lambda_a(x)\,, \qquad \overline{C}_y(x) \to \overline{C}_y(x) - \delta\zeta\lambda_y(x)\,,\\
&\psi(x) \to \psi(x) + \delta\zeta\Delta\psi(x)\,, \qquad \overline{\psi}(x) \to \overline{\psi}(x) + \delta\zeta\Delta\overline{\psi}(x)\,,\\
&\lambda_a(x) \to \lambda_a(x)\,, \qquad \lambda_y(x) \to \lambda_y(x)\,.
\end{aligned}
$$

The the changes in I_s and $\mathcal{L}_{WM}$ lead to

$$
\begin{aligned}
\int d^4x \Big\{ & \frac{\delta\Gamma^{[0]}}{\delta K_\mu^a(x)}\frac{\delta\Gamma^{[0]}}{\delta\widetilde{W}_a^\mu(x)} + \frac{\delta\Gamma^{[0]}}{\delta K_\mu^y(x)}\frac{\delta\Gamma^{[0]}}{\delta\widetilde{W}_y^\mu(x)} + \frac{\delta\Gamma^{[0]}}{\delta L_a(x)}\frac{\delta\Gamma^{[0]}}{\delta\widetilde{C}_a(x)}\\
&+\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{\nu}}_{L\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta n_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{e}}_{L\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta l_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{e}}_{R\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)}\\
&+\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{\nu}}_{L\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta n'_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{e}}_{L\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta l'_\alpha(x)} + \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{e}}_{R\alpha}(x)}\frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)}\\
&-\widetilde{\lambda}_a(x)\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_a(x)} - \widetilde{\lambda}_y(x) + \frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_y(x)} - \langle\Delta\mathcal{L}_{WM}(x)\rangle^{[0]} - \langle\Delta\mathcal{L}_{\psi m}(x)\rangle^{[0]}\Big\} = 0\,,
\end{aligned}
\tag{4.6}
$$

where

$$
\begin{aligned}
\langle\Delta\mathcal{L}_{WM}(x)\rangle^{[0]} &= \frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C]\Delta\mathcal{L}_{WM}(x)\exp\Big\{\mathrm{i}\big(I_{\text{eff}}^{[0]} + I_s\big)\Big\}\,,\\
\langle\Delta\mathcal{L}_{WM}(x)\rangle^{[0]} &= \frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C]\Delta\mathcal{L}_{\psi m}(x)\exp\Big\{\mathrm{i}\big(I_{\text{eff}}^{[0]} + I_s\big)\Big\}\,.
\end{aligned}
$$

With the definitions of $\Delta\mathcal{L}_{WM}(x)$ and $\Delta\mathcal{L}_{WM}(x)$

$$
\delta_B\mathcal{L}_{WM}(x) = \delta\zeta\Delta\mathcal{L}_{WM}(x)\,, \qquad \delta_B\mathcal{L}_{\psi m}(x) = \delta\zeta\Delta\mathcal{L}_{\psi m}(x)\,,
$$

one can write

$$
\begin{aligned}
\langle\Delta\mathcal{L}_{WM}(x)\rangle^{[0]} \;&=\; M^2\widetilde{W}_{a\mu}(x)\frac{\delta\Gamma^{[0]}}{\delta K_\mu^a(x)} + M^2\Big(\frac{g_1}{g}\Big)^2\widetilde{W}_{y\mu}(x)\frac{\delta\Gamma^{[0]}}{\delta K_\mu^y(x)}\\
&\quad -M^2\frac{g_1}{g}\widetilde{W}_{y\mu}(x)\frac{\delta\Gamma^{[0]}}{\delta K_\mu^3(x)} - M^2\frac{g_1}{g}\widetilde{W}_{3\mu}(x)\frac{\delta\Gamma^{[0]}}{\delta K_\mu^y(x)}\,,\\
\langle\Delta\mathcal{L}_{\psi m}(x)\rangle^{[0]} \;&=\; -m\frac{\delta\Gamma^{[0]}}{\delta l'_\alpha(x)}\widetilde{e}_{R\alpha}(x) + m\widetilde{\overline{e}}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)}\\
&\quad -m\frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)}\widetilde{e}_{L\alpha}(x) + m\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta l_\alpha(x)}\,.
\end{aligned}
$$

Next, from the invariance of the path integral of $\mathcal{Z}^{[0]}$ with respect to the translation of the integration variables $\overline{C}_a(x)$, $\overline{C}_y(x)$, $\lambda_a(x)$ and $\lambda_y(x)$, one can get a set of auxiliary identities

$$
\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_1(x)} - \partial_\mu\frac{\delta\Gamma^{[0]}}{\delta K_\mu^1(x)} - g_1\widetilde{W}_{y\mu}\frac{\delta\Gamma^{[0]}}{\delta K_\mu^2(x)} - g_1\widetilde{W}_{2\mu}\frac{\delta\Gamma^{[0]}}{\delta K_\mu^y(x)}
$$

$$+\frac{i}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{\nu}}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)}+\widetilde{\nu}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)}-\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta n_\alpha(x)}-\widetilde{e}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta n'_\alpha(x)}\Big\}=0\,, \tag{4.7}$$

$$\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_2(x)}-\partial_\mu\frac{\delta\Gamma^{[0]}}{\delta K^2_\mu(x)}+g_1\widetilde{W}_{y\mu}\frac{\delta\Gamma^{[0]}}{\delta K^1_\mu(x)}+g_1\widetilde{W}_{1\mu}\frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)}$$
$$+\frac{1}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{\nu}}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)}-\widetilde{\nu}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)}+\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta n_\alpha(x)}-\widetilde{e}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta n'_\alpha(x)}\Big\}=0\,, \tag{4.8}$$

$$\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_3(x)}-\partial_\mu\frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)}-g_1\widetilde{W}_{y\mu}\frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)}-g_1\widetilde{W}_{3\mu}\frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)}$$
$$+\frac{i}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta l_\alpha(x)}+\widetilde{e}_{R\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta l'_\alpha(x)}-\widetilde{\overline{e}}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p_\alpha(x)}-\widetilde{e}_{L\alpha}(x)\frac{\delta\Gamma^{[0]}}{\delta p'_\alpha(x)}\Big\}=0\,, \tag{4.9}$$

$$\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_y(x)}-\partial_\mu\frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)}-g\widetilde{W}_{y\mu}\frac{\delta\Gamma^{[0]}}{\delta K^3_\mu(x)}-g\widetilde{W}_{3\mu}\frac{\delta\Gamma^{[0]}}{\delta K^y_\mu(x)}=0\,, \tag{4.10}$$

and

$$\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}=\langle\Phi_a(x)\rangle^{[0]}\,,\qquad\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_y(x)}=\langle\Phi_y(x)\rangle^{[0]}\,. \tag{4.11}$$

where

$$\langle\Phi_a(x)\rangle^{[0]}=\frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\Phi_a(x)\exp\Big\{\mathrm{i}\big(I^{[0]}_{\rm eff}+I_s\big)\Big\}\,, \tag{4.12}$$
$$\langle\Phi_y(x)\rangle^{[0]}=\frac{1}{N\mathcal{Z}^{[0]}}\int\mathcal{D}[\overline{\psi},\psi,\mathcal{W},\overline{C},C,\lambda]\Phi_y(x)\exp\Big\{\mathrm{i}\big(I^{[0]}_{\rm eff}+I_s\big)\Big\}\,. \tag{4.13}$$

Let $\widetilde{\Phi}_a(x)$, $\widetilde{\Phi}_y(x)$, $\widetilde{\mathcal{L}}_{WM}$ and $\widetilde{\mathcal{L}}_{\psi m}$ be the results obtained from $\Phi_a(x)$, $\Phi_y(x)$, $\mathcal{L}_{WM}$ and $\mathcal{L}_{\psi m}$ by replacing the field functions with the classical field functions and define

$$\overline{\Gamma}^{[0]}=\Gamma^{[0]}-\int d^4x\Big\{\widetilde{\lambda}_a(x)\widetilde{\Phi}_a(x)+\widetilde{\lambda}_y(x)\widetilde{\Phi}_y(x)+\widetilde{\mathcal{L}}_{WM}+\widetilde{\mathcal{L}}_{\psi m}\Big\}\,, \tag{4.14}$$

Thus, from (4.6)–(4.11), one gets

$$\int d^4x\Big\{\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^a_\mu(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{W}^\mu_a(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{W}^\mu_y(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta L_a(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{C}_a(x)}$$
$$+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\nu}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta n_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{e}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta l_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{e}_{R\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta p_\alpha(x)}$$
$$+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{\nu}}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta n'_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{e}}_{L\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta l'_\alpha(x)}+\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{e}}_{R\alpha}(x)}\frac{\delta\overline{\Gamma}^{[0]}}{\delta p'_\alpha(x)}\Big\}=0\,. \tag{4.15}$$

and

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_a(x)}=\langle\Phi_a(x)\rangle^{[0]}-\widetilde{\Phi}_a(x)\,,\qquad\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_y(x)}=\langle\Phi_y(x)\rangle^{[0]}-\widetilde{\Phi}_y(x)\,, \tag{4.16}$$
$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_1(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^1_\mu(x)}-g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^2_\mu(x)}-g_1\widetilde{W}_{2\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}$$

$$+\frac{i}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{\nu}}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p_\alpha(x)}+\widetilde{\nu}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p'_\alpha(x)}-\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta n_\alpha(x)}-\widetilde{e}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta n'_\alpha(x)}\Big\}=0\,, \tag{4.17}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_2(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^2_\mu(x)}+g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^1_\mu(x)}+g_1\widetilde{W}_{1\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}$$
$$+\frac{1}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{\nu}}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p_\alpha(x)}-\widetilde{\nu}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p'_\alpha(x)}+\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta n_\alpha(x)}-\widetilde{e}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta n'_\alpha(x)}\Big\}=0\,, \tag{4.18}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_3(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^3_\mu(x)}-g_1\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^3_\mu(x)}-g_1\widetilde{W}_{3\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}$$
$$+\frac{i}{2}\frac{mg}{M^2}\Big\{\widetilde{\overline{e}}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta l_\alpha(x)}+\widetilde{e}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta l'_\alpha(x)}-\widetilde{\overline{e}}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p_\alpha(x)}-\widetilde{e}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}}{\delta p'_\alpha(x)}\Big\}=0\,, \tag{4.19}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\overline{C}}_y(x)}-\partial_\mu\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}-g\widetilde{W}_{y\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^3_\mu(x)}-g\widetilde{W}_{3\mu}\frac{\delta\overline{\Gamma}^{[0]}}{\delta K^y_\mu(x)}=0\,. \tag{4.20}$$

As mentioned earlier our intention to use the generalized form of the theory containing λ_a, λ_y and their sources is to study the Renormalisability of the theory for which such sources are absent from the generating functional for the Green functions and therefore $\langle\Phi_a(x)\rangle^{[0]}$ and $\langle\Phi_y(x)\rangle^{[0]}$ are equal to zero. We now, according to (4.11), let vanish $\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}$ and $\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}$ to make $\langle\Phi_a(x)\rangle^{[0]}$ and $\langle\Phi_y(x)\rangle^{[0]}$ equal to zero. This means

$$\widetilde{\Phi}_a(x)=0\,,\qquad \widetilde{\Phi}_y(x)=0\,, \tag{4.21}$$

and

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_a(x)}=0\,,\qquad \frac{\delta\overline{\Gamma}^{[0]}}{\delta\widetilde{\lambda}_y(x)}=0\,. \tag{4.22}$$

In the following we will denote by $\overline{\Gamma}^{[0]}[\psi,\overline{\psi},W,\overline{C},C,\lambda,K,L,n,l,p,n',l',p']$ the functional that is obtained from $\overline{\Gamma}^{[0]}[\widetilde{\psi},\widetilde{\overline{\psi}},\widetilde{W},\widetilde{\overline{C}},\widetilde{C},\widetilde{\lambda},K,\cdots]$ by replacing the classical field functions with the usual field functions. Assume that the dimensional regularization method is used and the Slavnov–Taylor identity and the auxiliary identities are guaranteed. Denote the tree part and one loop part of $\overline{\Gamma}^{[0]}$ by $\overline{\Gamma}^{[0]}_0$ and $\overline{\Gamma}^{[0]}_1$ respectively. $\overline{\Gamma}^{[0]}_0$ is thus the modified action $\overline{I}^{[0]}_{eff}$ obtained from $I^{[0]}_{eff}$ by excluding the mass term and (λ_a,λ_y) terms. From (4.15) and (4.17) – (4.22) one has

$$\Phi_a(x)=0\,,\qquad \Phi_y(x)=0\,, \tag{4.23}$$

$$\frac{\delta\overline{\Gamma}^{[0]}}{\delta\lambda_a(x)}=0\,,\qquad \frac{\delta\overline{\Gamma}^{[0]}}{\delta\lambda_y(x)}=0\,, \tag{4.24}$$

$$\Lambda_{op}\overline{\Gamma}^{[0]}_0=0\,,$$

and

$$\overline{\Gamma}^{[0]}_0*\overline{\Gamma}^{[0]}_1+\overline{\Gamma}^{[0]}_1*\overline{\Gamma}^{[0]}_0=\Lambda_{op}\overline{\Gamma}^{[0]}_1=0\,, \tag{4.25}$$

$$\Sigma_a(x)\overline{\Gamma}^{[0]} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[0]} = 0\,, \tag{4.26}$$

where Λ_{op},$\Sigma_a(x)$ and $\Sigma_y(x)$ are defined by

$$\begin{aligned}
\Lambda_{op} \;=\; & \int d^4x \Big\{ \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^a_\mu(x)}\frac{\delta}{\delta W^\mu_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_a(x)}\frac{\delta}{\delta K^a_\mu(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^y_\mu(x)}\frac{\delta}{\delta W^\mu_y(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_y(x)}\frac{\delta}{\delta K^y_\mu(x)} \\
&+\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta L_a(x)}\frac{\delta}{\delta C_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_a(x)}\frac{\delta}{\delta L_a(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \nu_{L\alpha}(x)}\frac{\delta}{\delta n_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n_\alpha(x)}\frac{\delta}{\delta \nu_{L\alpha}(x)} \\
&+\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{L\alpha}(x)}\frac{\delta}{\delta l_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l_\alpha(x)}\frac{\delta}{\delta e_{L\alpha}(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{R\alpha}(x)}\frac{\delta}{\delta p_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta p_\alpha(x)}\frac{\delta}{\delta e_{R\alpha}(x)} \\
&+\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \overline{\nu}_{L\alpha}(x)}\frac{\delta}{\delta n'_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n'_\alpha(x)}\frac{\delta}{\delta \overline{\nu}_{L\alpha}(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \overline{e}_{L\alpha}(x)}\frac{\delta}{\delta l'_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l'_\alpha(x)}\frac{\delta}{\delta \overline{e}_{L\alpha}(x)} \\
&+\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta \overline{e}_{R\alpha}(x)}\frac{\delta}{\delta p'_\alpha(x)} + \frac{\delta\overline{\Gamma}^{[0]}_0}{\delta p'_\alpha(x)}\frac{\delta}{\delta \overline{e}_{R\alpha}(x)} \Big\}\,,
\end{aligned} \tag{4.27}$$

$$\begin{aligned}
\Sigma_1(x) \;=\; & \frac{\delta}{\delta\overline{C}_1(x)} - \partial_\mu\frac{\delta}{\delta K^1_\mu(x)} - g_1 W_{y\mu}\frac{\delta}{\delta K^2_\mu(x)} - g_1 W_{2\mu}\frac{\delta}{\delta K^y_\mu(x)} \\
&+\frac{i}{2}\frac{mg}{M^2}\Big\{\overline{\nu}_{L\alpha}(x)\frac{\delta}{\delta p_\alpha(x)} + \nu_{L\alpha}(x)\frac{\delta}{\delta p'_\alpha(x)} - \overline{e}_{R\alpha}(x)\frac{\delta}{\delta n_\alpha(x)} - e_{R\alpha}(x)\frac{\delta}{\delta n'_\alpha(x)}\Big\} = 0\,,
\end{aligned} \tag{4.28}$$

$$\begin{aligned}
\Sigma_2(x) \;=\; & \frac{\delta}{\delta\overline{C}_2(x)} - \partial_\mu\frac{\delta}{\delta K^2_\mu(x)} + g_1 W_{y\mu}\frac{\delta}{\delta K^1_\mu(x)} + g_1 W_{1\mu}\frac{\delta}{\delta K^y_\mu(x)} \\
&+\frac{1}{2}\frac{mg}{M^2}\Big\{\overline{\nu}_{L\alpha}(x)\frac{\delta}{\delta p_\alpha(x)} - \nu_{L\alpha}(x)\frac{\delta}{\delta p'_\alpha(x)} + \overline{e}_{R\alpha}(x)\frac{\delta}{\delta n_\alpha(x)} - e_{R\alpha}(x)\frac{\delta}{\delta n'_\alpha(x)}\Big\} = 0\,,
\end{aligned} \tag{4.29}$$

$$\begin{aligned}
\Sigma_3(x) \;=\; & \frac{\delta}{\delta\overline{C}_3(x)} - \partial_\mu\frac{\delta}{\delta K^3_\mu(x)} - g_1 W_{y\mu}\frac{\delta}{\delta K^3_\mu(x)} - g_1 W_{3\mu}\frac{\delta}{\delta K^y_\mu(x)} \\
&+\frac{i}{2}\frac{mg}{M^2}\Big\{\overline{e}_{R\alpha}(x)\frac{\delta}{\delta l_\alpha(x)} + e_{R\alpha}(x)\frac{\delta}{\delta l'_\alpha(x)} - \overline{e}_{L\alpha}(x)\frac{\delta}{\delta p_\alpha(x)} - e_{L\alpha}(x)\frac{\delta}{\delta p'_\alpha(x)}\Big\} = 0\,,
\end{aligned} \tag{4.30}$$

$$\Sigma_y(x) \;=\; \frac{\delta}{\delta\overline{C}_y(x)} - \partial_\mu\frac{\delta}{\delta K^y_\mu(x)} - gW_{y\mu}\frac{\delta}{\delta K^3_\mu(x)} - gW_{3\mu}\frac{\delta}{\delta K^y_\mu(x)}\,. \tag{4.31}$$

The meaning of the notation $A*B$ is

$$\begin{aligned}
A*B = \int d^4x \Big\{ & \frac{\delta A}{\delta K^a_\mu(x)}\frac{\delta B}{\delta W^\mu_a(x)} + \frac{\delta A}{\delta K^y_\mu(x)}\frac{\delta B}{\delta W^\mu_y(x)} + \frac{\delta A}{\delta L_a(x)}\frac{\delta B}{\delta C_a(x)} \\
&+\frac{\delta A}{\delta \nu_{L\alpha}(x)}\frac{\delta B}{\delta n_\alpha(x)} + \frac{\delta A}{\delta e_{L\alpha}(x)}\frac{\delta B}{\delta l_\alpha(x)} + \frac{\delta A}{\delta e_{R\alpha}(x)}\frac{\delta B}{\delta p_\alpha(x)} \\
&+\frac{\delta A}{\delta \overline{\nu}_{L\alpha}(x)}\frac{\delta B}{\delta n'_\alpha(x)} + \frac{\delta A}{\delta \overline{e}_{L\alpha}(x)}\frac{\delta B}{\delta l'_\alpha(x)} + \frac{\delta A}{\delta \overline{e}_{R\alpha}(x)}\frac{\delta B}{\delta p'_\alpha(x)}\Big\}\,.
\end{aligned} \tag{4.32}$$

(4.24) – (4.26) are of course satisfied by the finite part and the pole part of $\overline{\Gamma}^{[0]}_1$. Thus the equations of the pole part $\overline{\Gamma}^{[0]}_1$ are

$$\frac{\delta\overline{\Gamma}^{[0]}_1}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[0]}_1}{\delta\lambda_y(x)} = 0\,, \tag{4.33}$$

$$\Lambda_{op}\overline{\Gamma}^{[0]}_1 = 0\,, \tag{4.34}$$

$$\Sigma_a(x)\overline{\Gamma}^{[0]}_1 = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[0]}_1 = 0\,. \tag{4.35}$$

It is known [3] that when $m=0$ the theory is renormalisable and $\overline{\Gamma}^{[0]}_{1,div}$ is a combination of 5 independent terms. Now one can also find the corresponding solutions of equations $(4.33)-(4.35)$. These solutions are as follows

$$T_{(1)}=T_{WL}-T_{GL}-T_{CK}+2I^{(C)}_m\,, \tag{4.36}$$

$$T_{(2)}=T_{WY}-T_{GY}-T_{CKY}\,, \tag{4.37}$$

$$T_{(3)}=T_{CK}+T_{CKY}+T_{nn'}+T_{ll'}+T_{pp'}\,, \tag{4.38}$$

$$T_{(4)}=T_{\nu L}+T_{eL}-T_{nn'}-T_{ll'}-I^{(C)}_m\,, \tag{4.39}$$

$$T_{(5)}=T_{eR}-T_{pp'}-I^{(C)}_m\,, \tag{4.40}$$

where

$$T_{GL}=g\frac{\partial\overline{\Gamma}^{[0]}_0}{\partial g}\,,\quad T_{GY}=g_1\frac{\partial\overline{\Gamma}^{[0]}_0}{\partial g_1}\,,\quad I^{(C)}_m=m\frac{\partial\overline{\Gamma}^{[0]}_0}{\partial m}=u\frac{\partial\overline{\Gamma}^{[0]}_0}{\partial u}\,,$$

$$T_{WL}=\int d^4x\Big\{W^\mu_a(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_a(x)}+L_a(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta L_a(x)}\Big\}\,,$$

$$T_{WY}=\int d^4xW^\mu_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta W^\mu_y(x)}\,,$$

$$T_{CK}=\int d^4x\Big\{\overline{C}_a(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\overline{C}_a(x)}+C_a(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_a(x)}+K^a_\mu(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^a_\mu(x)}\Big\}\,,$$

$$T_{CKY}=\int d^4x\Big\{\overline{C}_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\overline{C}_(x)}+C_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_y(x)}+K^y_\mu(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta K^y_\mu(x)}\Big\}\,,$$

$$T_{\nu L}=\int d^4x\Big\{\nu_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\nu_{L\alpha}(x)}+\overline{\nu}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\overline{\nu}_{L\alpha}(x)}\Big\}\,,$$

$$T_{eL}=\int d^4x\Big\{e_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{L\alpha}(x)}+\overline{e}_{L\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\overline{e}_{L\alpha}(x)}\Big\}\,,$$

$$T_{eR}=\int d^4x\Big\{e_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta e_{R\alpha}(x)}+\overline{e}_{R\alpha}(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta\overline{e}_{R\alpha}(x)}\Big\}\,,$$

$$T_{nn'}=\int d^4x\Big\{n_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n_\alpha(x)}+n'_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta n'_\alpha(x)}\Big\}\,,$$

$$T_{ll'}=\int d^4x\Big\{l_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l_\alpha(x)}+l'_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta l'_\alpha(x)}\Big\}\,,$$

$$T_{pp'}=\int d^4x\Big\{p_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta p_\alpha(x)}+p'_\alpha(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta p'_\alpha(x)}\Big\}\,,$$

where the parameter u appearing in the expression of $I^{(C)}_m$ stands for m/M^2. Similar to the case of Ref. [3], $T_{(3)}$ is $2\big(\overline{\Gamma}^{[0]}_0-I_{WL}-I_{WY}-I_\psi-I_{\psi W}\big)$. $T_{(1)}$ is a combination of I_{WL}, $T_{(3)}$ and $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_y(x)}$. $T_{(2)}$ is a combination of I_{WY} and $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_y(x)}$. The sum of $T_{(4)}$ and $T_{(5)}$ is $2\big(I_\psi+I_{\psi W}\big)$. $\int d^4xC_y(x)\frac{\delta\overline{\Gamma}^{[0]}_0}{\delta C_y(x)}$ and $T_{(5)}$ can be easily checked to satisfy $(4.33)-(4.35)$.

Since (4.36)−(4.40) become the whole independent terms of $\overline{\Gamma}^{[0]}_{1,div}$ when $m = 0$, a new term that appears when $m \neq 0$ should include $m\ \overline{\psi}_\alpha\ \psi_{\alpha'}$ as a factor and also satisfies (4.33) – (4.35). First, it is clearly not possible to form such a solution of (4.33) – (4.35) if negative dimension coefficients are excluded. Next, if M^2 is included as a factor in the definition of the ghost action so that the ghost fields become dimensionless, then the modified effective action (without λ terms) does not contain negative dimension coefficients. Thus a new term that can appear in $\overline{\Gamma}^{[0]}_1$ must be formed with some powers of $\overline{C_\sigma}\ C_{\sigma'}$ and a factor from $m\overline{\psi}_\alpha\psi_{\alpha'}$, where σ and σ' stand for $1,2,3,y$. However, (4.33) – (4.35) does not have such a solution neither. This can easily be seen from (4.35). It follows that $\overline{\Gamma}^{[0]}_{1,div}$ is a combination of (4.36)–(4.40). Namely

$$\overline{\Gamma}^{[0]}_{1,div} = \alpha^{(1)}_1 T_{(1)} + \alpha^{(1)}_2 T_{(2)} + \alpha^{(1)}_3 T_{(3)} + \alpha^{(1)}_4 T_{(4)} + \alpha^{(1)}_5 T_{(5)}\,, \tag{4.41}$$

where, $\alpha^{(1)}_1, \cdots, \alpha^{(1)}_5$ are constants of order $(\hbar)^1$ and are divergent when the space-time dimension tends to 4.

In order to cancel the one loop divergence the counterterm of order $\hbar^1$ in the action should be chosen as

$$\delta I^{[1]}_{count} = -\overline{\Gamma}^{[0]}_{1,div}\,. \tag{4.42}$$

Since

$$\overline{I}^{[0]}_{eff} = \overline{\Gamma}^{[0]}_0\,, \tag{4.43}$$

it is known from (4.41) that the sum of $\overline{I}^{[0]}_{eff}$ and $\delta I^{[1]}_{count}$, to order of $\hbar^1$, can be written as

$$\begin{aligned}&\overline{I}^{[1]}_{eff}[\psi,\overline{\psi},W,C,\overline{C},K,L,n,l,p,n',l',p',g,g_1,u]\\ &\quad = \overline{I}^{[0]}_{\rm eff}[\psi^{[0]},\overline{\psi}^{[0]},W^{[0]},C^{[0]},\overline{C}^{[0]},K^{[0]},L^{[0]},n^{[0]},{n'}^{[0]},\cdots,g^{[0]},g^{[0]}_1,u^{[0]}]\,,\end{aligned} \tag{4.44}$$

where the bare fields and the bare parameters (to order $(\hbar)^1$) are defined as

$$W^{[0]}_{a\mu} = (Z^{[1]}_3)^{1/2} W_{a\mu} = \big(1-\alpha^{(1)}_1\big) W_{a\mu}\,, \qquad L^{[0]}_a = (Z^{[1]}_3)^{1/2} L_a\,, \tag{4.45}$$

$$W^{[0]}_{y\mu} = ({Z'}^{[1]}_3)^{1/2} W_{y\mu} = \big(1-\alpha^{(1)}_2\big) W_{y\mu}\,, \tag{4.46}$$

$$C^{[0]}_a = (\widetilde{Z}^{[1]}_3)^{1/2} C_a = \big(1-\alpha^{(1)}_3+\alpha^{(1)}_1\big) C_a\,, \tag{4.47}$$

$$\overline{C}^{[0]}_a = (\widetilde{Z}^{[1]}_3)^{1/2}\overline{C}_a\,, \qquad K^{a[0]}_\mu = (\widetilde{Z}^{[1]}_3)^{1/2} K^a_\mu\,, \tag{4.48}$$

$$C^{[0]}_y = (\widetilde{Z}'^{[1]}_3)^{1/2} C_y = \big(1-\alpha^{(1)}_3+\alpha^{(1)}_2\big) C_y\,, \tag{4.49}$$

$$\overline{C}^{[0]}_y = (\widetilde{Z}'^{[1]}_3)^{1/2}\overline{C}_y\,, \qquad K^{y[0]}_\mu = (\widetilde{Z}^{[1]}_3)^{1/2} K^y_\mu\,, \tag{4.50}$$

$$\nu_L^{[0]} = (Z_{\nu L}^{[1]})^{1/2}\nu_L = \big(1-\alpha_4^{(1)}\big)\nu_L\,, \quad \overline{\nu}_L^{[0]} = (Z_{\nu L}^{[1]})^{1/2}\overline{\nu}_L\,, \tag{4.51}$$

$$e_L^{[0]} = (Z_{eL}^{[1]})^{1/2}e_L = (Z_{\nu L}^{[1]})^{1/2}e_L\,, \quad \overline{e}_L^{[0]} = (Z_{eL}^{[1]})^{1/2}\overline{e}_L\,, \tag{4.52}$$

$$e_R^{[0]} = (Z_{eR}^{[1]})^{1/2}e_R = \big(1-\alpha_5^{(1)}\big)e_R\,, \quad \overline{e}_R^{[0]} = (Z_{eR}^{[1]})^{1/2}\overline{e}_R\,, \tag{4.53}$$

$$n^{[0]} = (Z_{(n)}^{[1]})^{1/2}n = \big(1-\alpha_3^{(1)}+\alpha_4^{(1)}\big)n\,, \quad n'^{[0]} = (Z_{(n)}^{[1]})^{1/2}n'\,, \tag{4.54}$$

$$l^{[0]} = (Z_{(l)}^{[1]})^{1/2}l = (Z_{(n)}^{[1]})^{1/2}l\,, \quad l'^{[0]} = (Z_{(l)}^{[1]})^{1/2}l'\,, \tag{4.55}$$

$$p^{[0]} = (Z_{(p)}^{[1]})^{1/2}p = \big(1-\alpha_3^{(1)}+\alpha_5^{(1)}\big)p\,, \quad p'^{[0]} = (Z_{(p)}^{[1]})^{1/2}p'\,, \tag{4.56}$$

$$g^{[0]} = Z_g^{[1]}g = (Z_3^{[1]})^{-1/2}g\,, \quad g_1^{[0]} = Z_g'^{[1]}g_1 = (Z_3'^{[1]})^{-1/2}g_1\,, \tag{4.57}$$

$$u^{[0]} = \big(1-2\alpha_1^{(1)}+\alpha_4^{(1)}+\alpha_5^{(1)}\big)u\,. \tag{4.58}$$

Next let $\Phi_a^{[0]}$ and $\Phi_y^{[0]}$ be obtained from Φ_a and Φ_y by replacing the field functions and parameters with the bare field functions and bare parameters. From (4.45), (4.46) and (4.57) one has

$$\Phi_a^{[0]} = (Z_3^{[1]})^{1/2}\Phi_a\,, \quad \Phi_y^{[0]} = (Z_3'^{[1]})^{1/2}\Phi_y\,. \tag{4.59}$$

Thus by adding the mass terms and the λ terms into $\overline{I}_{eff}^{[1]}$ and forming

$$I_{eff}^{[1]} = \overline{I}_{eff}^{[1]} + I_{WM} + I_{\psi m} + \int d^4x\Big\{\lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x)\Big\}\,, \tag{4.60}$$

one gets

$$\begin{aligned} &I_{eff}^{[1]}[\psi,\overline{\psi},W,C,\overline{C},\lambda,K,L,n,l,p,n',l',p',g,g_1,M,m] \\ &\quad = I_{eff}^{[0]}[\psi^{[0]},\overline{\psi}^{[0]},W^{[0]},C^{[0]},\overline{C}^{[0]},\lambda^{[0]},K^{[0]},L^{[0]},n^{[0]},n'^{[0]},\cdots,g^{[0]},g_1^{[0]},M^{[0]},m^{[0]}]\,, \end{aligned} \tag{4.61}$$

where

$$M^{[0]} = (Z_3^{[1]})^{-1/2}M\,, \quad m^{[0]} = (Z_{eL}^{[1]})^{-1/2}(Z_{eR}^{[1]})^{-1/2}m\,, \tag{4.62}$$

and

$$\lambda_a^{[0]} = (Z_3^{[1]})^{-1/2}\lambda_a\,, \quad \lambda_y^{[0]} = (Z_3'^{[1]})^{-1/2}\lambda_y\,. \tag{4.63}$$

Obviously, if the action $I_{eff}^{[1]}$ is used to replace $I_{eff}^{[0]}$ in (4.3) and define $\mathcal{Z}^{[1]}$, $\Gamma^{[1]}$ as well as

$$\overline{\Gamma}^{[1]} = \Gamma^{[1]} - I_{WM} - I_{\psi m} - \int d^4x\Big\{\lambda_a(x)\Phi_a(x) + \lambda_y(x)\Phi_y(x)\Big\}\,, \tag{4.64}$$

then one has

$$\begin{aligned} &\overline{\Gamma}^{[1]}[\psi,\overline{\psi},W,C,\overline{C},\lambda,K,L,n,l,p,n',l',p',g,g_1,u] \\ &\quad = \overline{\Gamma}^{[0]}[\psi^{[0]},\overline{\psi}^{[0]},W^{[0]},C^{[0]},\overline{C}^{[0]},\lambda^{[0]},K^{[0]},L^{[0]},n^{[0]},n'^{[0]},\cdots,g^{[0]},g_1^{[0]},u^{[0]}]\,. \end{aligned} \tag{4.65}$$

From this it is easy to check that, to order $\hbar^1$, $\overline{\Gamma}^{[1]}$ is finite. Moreover, by changing into bare fields and bare parameters the fields and parameters in (4.15)– (4.22) and then transforming them back into the renormalized fields and renormalized parameters according to (4.45)−(4.59), one can see that, under condition (4.23), $\overline{\Gamma}^{[1]}$ also satisfies

$$\Lambda_{op}\overline{\Gamma}^{[1]} = 0\,, \tag{4.66}$$

$$\frac{\delta\overline{\Gamma}^{[1]}}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[1]}}{\delta\lambda_y(x)} = 0\,, \tag{4.67}$$

$$\Sigma_a(x)\overline{\Gamma}^{[1]} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[1]} = 0\,. \tag{4.68}$$

We can now use the inductive method and follow the steps of Ref. [3] to complete the proof of renormalisability. Assume that up to n loop the theory has been proved to be renormalisable by introducing the counterterm

$$I^{[n]}_{\text{count}} = \sum_{l=1}^{n} \delta I^{[l]}_{\text{count}},$$

where $\delta I^{[l]}_{count}$ is the counterterm of order $\hbar^l$ and has the form of (4.41),(4.42). Therefore the modified generating functional $\overline{\Gamma}^{[n]}$ for the regular vertex, defined by the action

$$I^{[n]}_{\text{eff}} = I^{[0]}_{\text{eff}} + I^{[n]}_{\text{count}},$$

satisfied equations (4.66) – (4.68) (under (4.23)) and, to order $\hbar^n$, is finite. This also means that the fields or parameters in each of the following brackets have the same renormalization factor

$$(W^{[0]}_{a\mu}, L_a), (C_a, \overline{C}_a, K^a_\mu), (C_y, \overline{C}_y, K^y_\mu), (\nu_L, \overline{\nu}_L, e_L, \overline{e}_L), (e_R, \overline{e}_R), (n, n', l, l'), (p, p'), (\lambda, M, g),$$

and that

$$Z_g^{'[n]}(Z_3^{'[n]})^{1/2} = 1\,, \qquad Z_g^{[n]}(Z_3^{[n]})^{1/2} = 1\,,$$
$$Z_3^{[n]}\widetilde{Z}_3^{[n]} = \widetilde{Z}_3^{'[n]}\widetilde{Z}_3^{'[n]} = Z^{[n]}_{\nu L}Z^{[n]}_{(n)} = Z^{[n]}_{eR}Z^{[n]}_{(p)}\,.$$

Denote by $\overline{\Gamma}^{[n]}_k$ the part of order $\hbar^k$ in $\overline{\Gamma}^{[n]}$. For $k \le n$, $\overline{\Gamma}^{[n]}_k$ is equal to $\overline{\Gamma}^{[k]}_k$, because it can not contain the contribution of a counterterm of order $\hbar^{k+1}$ or higher. Thus on expanding $\overline{\Gamma}^{[n]}$ to order $\hbar^{n+1}$ one has

$$\overline{\Gamma}^{[n]} = \sum_{k=0}^{n} \overline{\Gamma}^{[k]}_k + \overline{\Gamma}^{[n]}_{n+1} + \cdots\,.$$

Using this and extracting the terms of order $\hbar^{(n+1)}$ from the equations satisfied by $\overline{\Gamma}^{[n]}$, namely (4.66) – (4.68), one finds

$$\Lambda_{op}\overline{\Gamma}^{[n]}_{n+1} = 0\,, \tag{4.69}$$

$$\frac{\delta\overline{\Gamma}^{[n]}_{n+1}}{\delta\lambda_a(x)} = 0\,, \qquad \frac{\delta\overline{\Gamma}^{[n]}_{n+1}}{\delta\lambda_y(x)} = 0\,, \tag{4.70}$$

$$\Sigma_a(x)\overline{\Gamma}^{[n]}_{n+1} = 0\,, \qquad \Sigma_y(x)\overline{\Gamma}^{[n]}_{n+1} = 0\,. \tag{4.71}$$

Let $\overline{\Gamma}^{[n]}_{n+1,div}$ stand for the pole part of $\overline{\Gamma}^{[n]}_{n+1}$. By repeating the steps going from (4.33) to (4.41), one can arrive at

$$\overline{\Gamma}^{[n]}_{n+1,div} = \alpha_1^{(n+1)}T_{(1)} + \alpha_2^{(n+1)}T_{(2)} + \alpha_3^{(n+1)}T_{(3)} + \alpha_4^{(n+1)}T_{(4)} + \alpha_5^{(n+1)}T_{(5)}\,, \tag{4.72}$$

where $\alpha_1^{(n+1)}, \cdots, \alpha_5^{(n+1)}$ are constants of order $(\hbar)^{n+1}$. Therefore, in order to cancel the $n+1$ loop divergence the counterterm of order $\hbar^{n+1}$ should be chosen as

$$\delta I^{[n+1]}_{count} = -\overline{\Gamma}^{[n]}_{n+1,div}[\psi,\overline{\psi},W,C,\overline{C}]\,. \tag{4.73}$$

Adding this counterterm, the mass term and the λ terms to $\overline{I}^{[n]}_{\rm eff}$, one can express the effective action of order $\hbar^{n+1}$ as

$$\begin{aligned} &I^{[n+1]}_{\rm eff}[\psi,\overline{\psi},W,C,\overline{C},\lambda,K,L,n,l,p,n',l',p',g,g_1,M,m] \\ &\quad = I^{[0]}_{\rm eff}[\psi^{[0]},\overline{\psi}^{[0]},W^{[0]},C^{[0]},\overline{C}^{[0]},\lambda^{[0]},K^{[0]},L^{[0]},n^{[0]},n^{'[0]},\cdots,g^{[0]},g_1^{[0]},M^{[0]},m^{[0]}]\,, \end{aligned} \tag{4.74}$$

where the bare fields and the bare parameters (to order $(\hbar)^{n+1}$) are defined as

$$W^{[0]}_{a\mu} = (Z_3^{[n+1]})^{1/2}W_{a\mu} = \big((Z_3^{[n]})^{1/2} - \alpha_1^{(n+1)}\big)W_{a\mu}\,, \quad L^{[0]}_a = (Z_3^{[n+1]})^{1/2}L_a\,, \tag{4.75}$$

$$W^{[0]}_{y\mu} = (Z_3^{'[n+1]})^{1/2}W_{y\mu} = \big((Z_3^{'[n]})^{1/2} - \alpha_2^{(n+1)}\big)W_{y\mu}\,, \tag{4.76}$$

$$C^{[0]}_a = (\widetilde{Z}_3^{[n+1]})^{1/2}C_a = \big((\widetilde{Z}_3^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_1^{(n+1)})\big)C_a\,, \tag{4.77}$$

$$\overline{C}^{[0]}_a = (\widetilde{Z}_3^{[n+1]})^{1/2}\overline{C}_a\,, \qquad K^{a[0]}_\mu = (\widetilde{Z}_3^{[n+1]})^{1/2}K^a_\mu\,, \tag{4.78}$$

$$C^{[0]}_y = (\widetilde{Z}_3^{'[n+1]})^{1/2}C_y = \big((\widetilde{Z}_3^{'[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_2^{(n+1)})\big)C_y\,, \tag{4.79}$$

$$\overline{C}^{[0]}_y = (\widetilde{Z}_3^{'[n+1]})^{1/2}\overline{C}_y\,, \qquad K^{y[0]}_\mu = (\widetilde{Z}_3^{[n+1]})^{1/2}K^y_\mu\,, \tag{4.80}$$

$$\nu^{[0]}_L = (Z_{\nu L}^{[n+1]})^{1/2}\nu_L = \big((Z_{\nu L}^{[n]})^{1/2} - \alpha_4^{(n+1)}\big)\nu_L\,, \quad \overline{\nu}^{[0]}_L = (Z_{\nu L}^{[n+1]})^{1/2}\overline{\nu}_L\,, \tag{4.81}$$

$$e^{[0]}_L = (Z_{eL}^{[n+1]})^{1/2}e_L = (Z_{\nu L}^{[n+1]})^{1/2}e_L\,, \quad \overline{e}^{[0]}_L = (Z_{eL}^{[n+1]})^{1/2}\overline{e}_L\,, \tag{4.82}$$

$$e^{[0]}_R = (Z_{eR}^{[n+1]})^{1/2}e_R = \big((Z_{eR}^{[n]})^{1/2} - \alpha_5^{(n+1)}\big)e_R\,, \quad \overline{e}^{[0]}_R = (Z_{eR}^{[n+1]})^{1/2}\overline{e}_R\,, \tag{4.83}$$

$$n^{[0]} = (Z_{(n)}^{[n+1]})^{1/2}n = \big((Z_{(n)}^{[n]})^{1/2} + (-\alpha_3^{(n+1)} + \alpha_4^{(n+1)})\big)n\,, \quad n^{'[0]} = (Z_{(n)}^{[n+1]})^{1/2}n'\,, \tag{4.84}$$

$$l^{[0]} = (Z_{(l)}^{[n+1]})^{1/2}l = (Z_{(n)}^{[n+1]})^{1/2}l\,, \quad l^{'[0]} = (Z_{(l)}^{[n+1]})^{1/2}l'\,, \tag{4.85}$$

$$p^{[0]} = (Z_{(p)}^{[n+1]})^{1/2}p = \big((Z_{(p)}^{[n]})^{1/2} - \alpha_3^{(n+1)} + \alpha_5^{(n+1)}\big)p\,, \quad p^{'[0]} = (Z_{(p)}^{[n+1]})^{1/2}p'\,, \tag{4.86}$$

$$g^{[0]} = Z_g^{[n+1]}g = (Z_3^{[n+1]})^{-1/2}g\,, \quad g_1^{[0]} = Z_g^{'[n+1]}g_1 = (Z_3^{'[n+1]})^{-1/2}g_1\,, \tag{4.87}$$

$$g^{[0]} = Z_g^{[n+1]} g = (Z_3^{[n+1]})^{-1/2} g\,, \quad g_1^{[0]} = Z_g^{'[n+1]} g_1 = (Z_3^{'[n+1]})^{-1/2} g_1\,, \tag{4.88}$$

$$M^{[0]} = Z_M^{[n+1]} M = (Z_3^{[n+1]})^{-1/2} M\,, \quad m^{[0]} = Z_m^{[n+1]} m = (Z_{eL}^{[n+1]})^{-1/2} (Z_{eR}^{[n+1]})^{-1/2} M\,, \tag{4.89}$$

and $\lambda_a^{[0]}, \lambda_y^{[0]}$ are

$$\lambda_a^{[0]} = (Z_3^{[n+1]})^{-1/2} \lambda_a\,, \quad \lambda_y^{[0]} = (Z_3^{'[n+1]})^{-1/2} \lambda_y\,. \tag{4.90}$$

Therefore, in terms of such bare fields and bare parameters, $\overline{\Gamma}^{[n+1]}$ can be expressed as

$$\begin{aligned} &\overline{\Gamma}^{[n+1]}[W, C, \overline{C}, \psi, \overline{\psi}, K, L, n, l, p, n', l', p', g, g_1, M] \\ &\quad = \widehat{\Gamma}^{[0]}[W^{[0]}, C^{[0]}, \overline{C}^{[0]}, \psi^{[0]}, \overline{\psi}^{[0]}, K^{[0]}, L^{[0]}, n^{[0]}, n^{'[0]}, \cdots, g^{[0]}, g_1^{[0]}, M^{[0]}]\,. \end{aligned} \tag{4.91}$$

From this one can conclude that $\overline{\Gamma}^{[n+1]}$, under (4.23), satisfies (4.66)$-$(4.68) and is finite to order $\hbar^{n+1}$. That is to say the theory is renormalisable.

V. Concluding Remarks

We have expounded that $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ electroweak theory with massive W Z fields and massive electron fields can still be quantized in a way similar to that used in Ref. [3] by taking into account the constraint conditions caused by these mass terms and the additional condition chosen by us. We have also shown that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the constraint conditions are rewritten as Fourier integrals with Lagrange multipliers λ_a and λ_y, the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields is BRST invariant. Furthermore, with the help of the renormalisability of the theory without the the mass term of matter fields we have found the general form of the divergent part of the generating functional Γ and proven that the mass term of the electron fields is also harmless to the renormalisability of the theory.

It is worth while emphasizing the following special features of the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ electroweak theory with massive W Z fields and massive electron fields. (1) These mass terms do not appear in the divergent part of Γ. (2) The ghost–electron coupling term $I_m^{(C)}$, which is caused by the mass term of the electron fields and contains the negative dimension parameter m/M^2, is not an independent term of the divergent part of Γ. If this were not the case, the mass terms would be harmful to the renormalisability of the theory.

As pointed out in Ref. [3], since the whereabouts of the Higgs Bosons is still unknown, it is reasonable to ask if the successes of the standard model of the electroweak theory really depends on the Higgs mechanism and to pay attention to the theory without the Higgs mechanism.

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